Using Progressions to Understand the Common Core Fraction Standards

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MaThink 2015

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Ice Breaker

- Arrange yourselves in a circle in order of birthday
  - **Month** and **Day**

Food for thought…

- *Does anyone share a birthday?*
- *What is the probability that at least two people would share a birthday in a group this size?*

Who are we?

Zachary and Jennifer

- Our presentation and handouts will be available on the MaThink wiki site.
- Our contact information:
  - Zachary.Hagman “at” gmail.com
  - Jennifer “at” mathandteaching.org
- We are math nerds.
- We take foot photos … a lot of foot photos.

Our Focus

Through activities and discussion, participants will gain a better understanding of the Common Core Progressions in general. More specifically, participants will delve into the progressions for grades 3-5 on fractions and grades 6-7 on ratio and proportional reasoning to see the purpose of the progressions and how they will be of great use to a teacher as she/he plans lessons. In addition, uses for a number line as a tool when teaching these concepts will be discussed. Questions from the participants regarding the CC Progressions will be answered and resources highlighted.

Progressions

The Common Core State Standards in mathematics were built on progressions, that is, narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. These documents trace major concepts through the grade levels. They explain which standards build upon one another, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics.

From: http://ime.math.arizona.edu/progressions/

Progressions

K-12 Domains

<table>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td colspan="9">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td></tr>
<tr><td colspan="2">Counting and Cardinality</td><td colspan="3">Number and Operations---Fractions</td><td colspan="2">Ratios and Proportional Relationships</td><td>Functions</td></tr>
</table>

High School Conceptual Categories

NUMBER AND QUANTITY	ALGEBRA	FUNCTIONS	GEOMETRY	STATISTICS AND PROBABILITY

From: http://www.illustrativemathematics.org/

A Video!

From: *http://threeacts.mrmeyer.com/nana/*

Do you need more information?

- Nana likes 1 cup of milk with 4 scoops of chocolate.
- Dan added 1 cups of milk with 5 scoops of chocolate.
- Can Dan fix his mistake? If so, how?
- Dan doesn't have enough milk to start over.
- The glass won't hold 2 cups of milk.

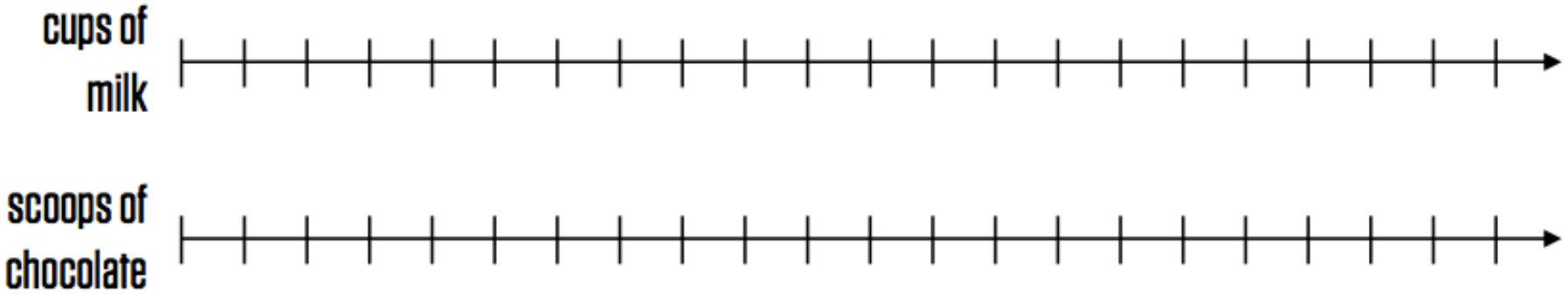
Let's discuss.

- What did you do to solve the problem?
- What tools did you use?
- What math did you encounter?
- Is a number line useful in this situation?
- For what grade level is this problem appropriate?

How did Dan solve the problem?

A Use for Number Lines

- How could we use a number line to solve the previous problem?

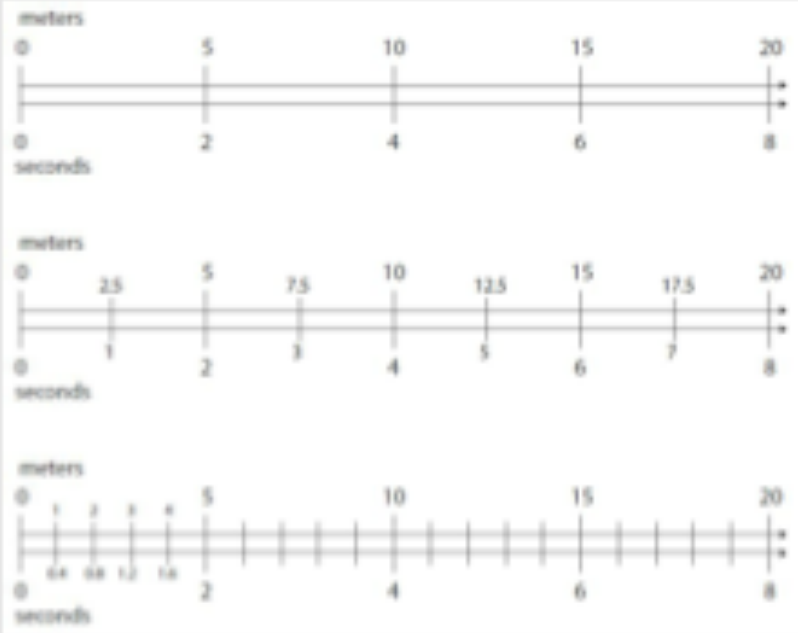


Relation to the Progressions

"Although it is traditional to move students quickly to solving proportions by setting up an equation, the Standards do not require this method in Grade 6" (pg 6).

"The study of proportional relationships is a foundation for the study of functions, which continues through High School and beyond" (pg 11).

Double number line diagrams used for situations with different units



Double number lines indicate coordinated multiplying and dividing of quantities. This can also be indicated in tables.

Format of the Progressions

- On the Left: You see the narrative.
- On the Right: The Standards discussed and problems, explanations, and alternative ways to approach problems.

Multiplying and dividing fractions In Grade 4 students connected fractions with addition and multiplication, understanding that

$$\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 5 \times \frac{1}{3}.$$

In Grade 5, they connect fractions with division, understanding that

$$5 \div 3 = \frac{5}{3},$$

or, more generally, $\frac{a}{b} = a \div b$ for whole numbers a and b, with b not equal to zero.[5.NF.3] They can explain this by working with their understanding of division as equal sharing (see figure in margin). They also create story contexts to represent problems involving division of whole numbers. For example, they see that

> If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get?

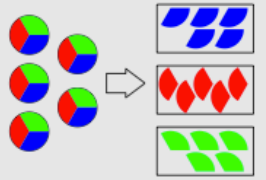
can be solved in two ways. First, they might partition each pound among the 9 people, so that each person gets $50 \times \frac{1}{9} = \frac{50}{9}$ pounds. Second, they might use the equation $9 \times 5 = 45$ to see that each person can be given 5 pounds, with 5 pounds remaining. Partitioning the remainder gives $5\frac{5}{9}$ pounds for each person.

Students have, since Grade 1, been using language such as "third of" to describe one part when a whole is partitioned into three parts. With their new understanding of the connection between fractions and division, students now see that $\frac{5}{3}$ is one third of 5, which leads to the meaning of multiplication by a unit fraction:

$$\frac{1}{3} \times 5 = \frac{5}{3}.$$

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

How to share 5 objects equally among 3 shares:
$5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$



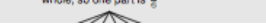
If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.

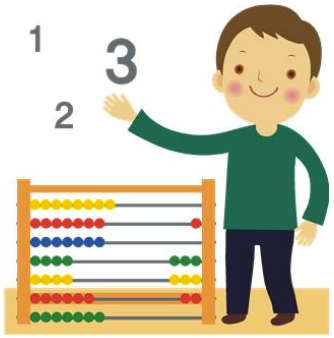
5.NF.4a Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

Using a fraction strip to show that $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

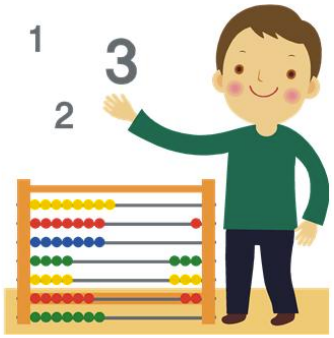
(c) 6 parts make one whole, so one part is $\frac{1}{6}$





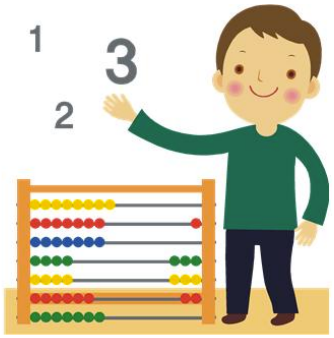
Another problem.

- Consider the 4 by 10 grid below. Shade any 6 units.
- Use the picture to find the percent of the grid that is shaded.



Discuss

- How did you solve the problem?
- How did you use the picture?
- Do students need to know how to convert fractions to decimals before attempting the problem? Why or why not?
- Would a number line be useful?



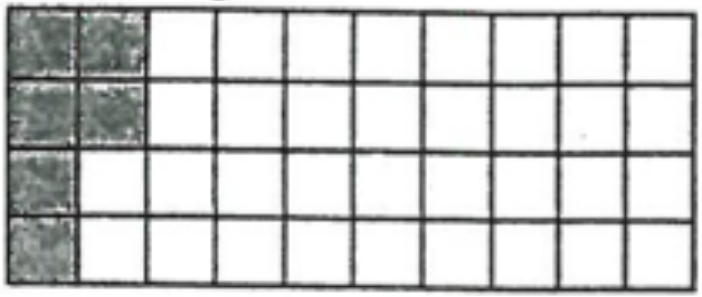
The Case of Ron Castleman

Some possible solutions:

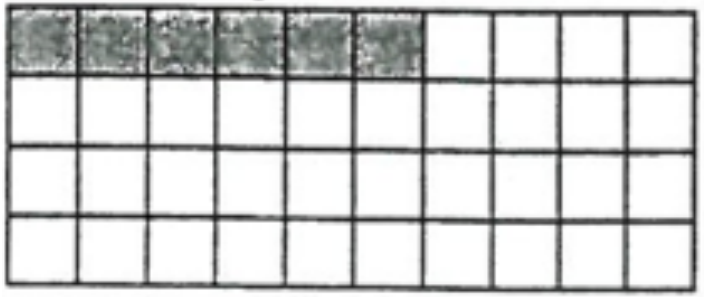
- What percent does each column represent?

- What percent does each row represent?

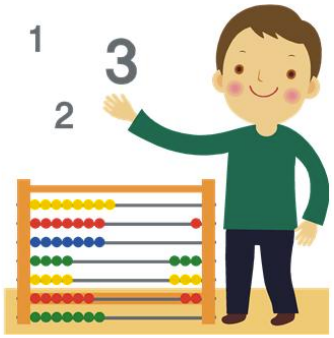
First Configuration



Second Configuration

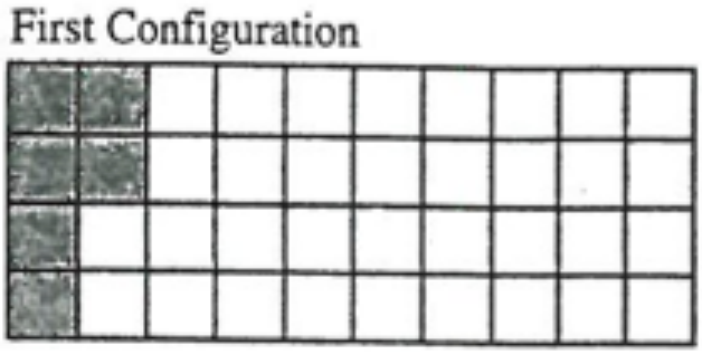


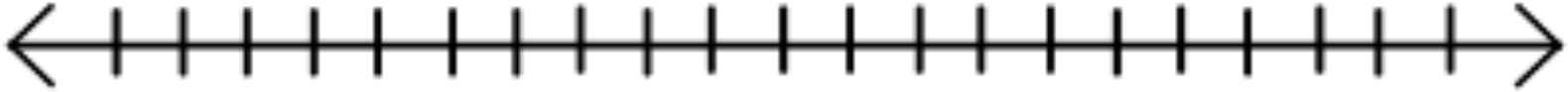
From: Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development, pgs 37-63



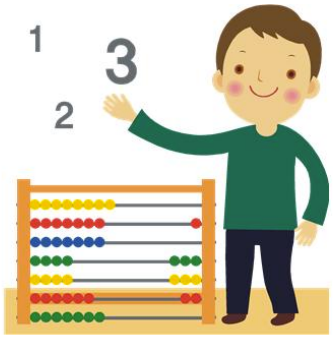
The Case of Ron Castleman

How can a number line be used?





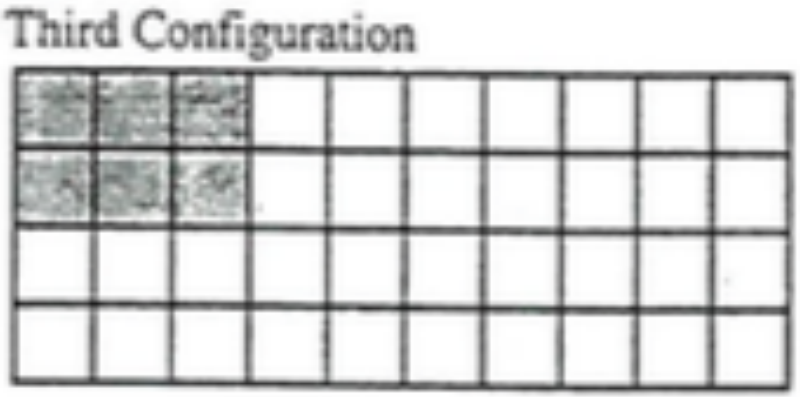
From: Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development, pgs 37-63



The Case of Ron Castleman

Another possible solution:

- Describe how this student could have solved the problem.



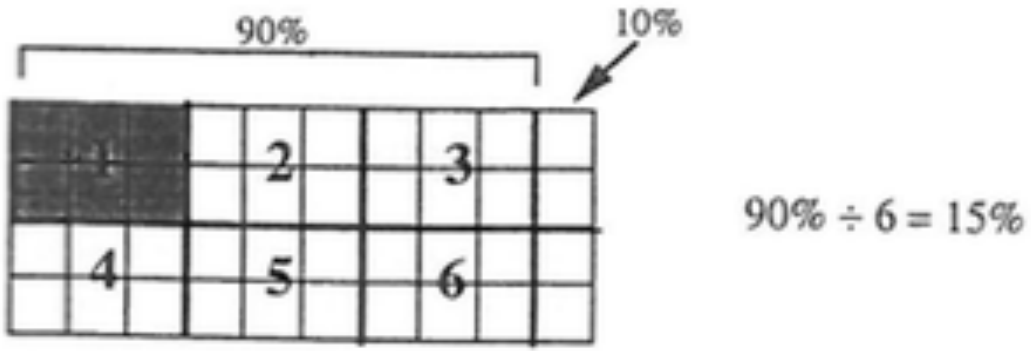
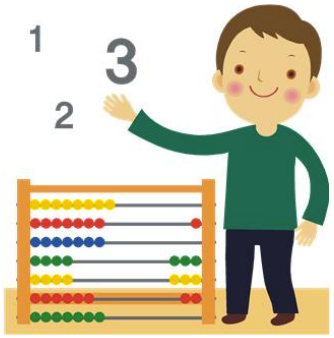


FIGURE 5.4. The diagram displayed by Omar and Marcus.

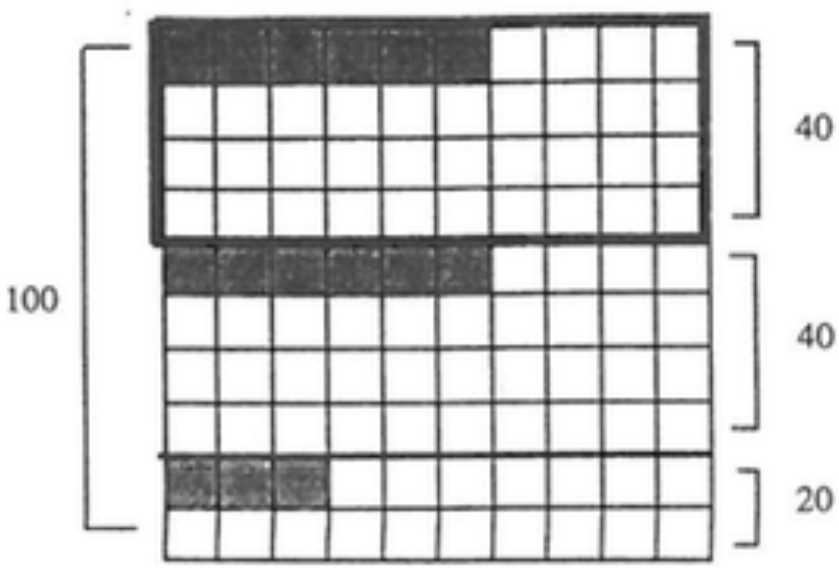
From: Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development, pgs 37-63



The Case of Ron Castleman

Yet, another possible solution:

- What did this student do?
- Compare and contrast this method with the others we have discussed.



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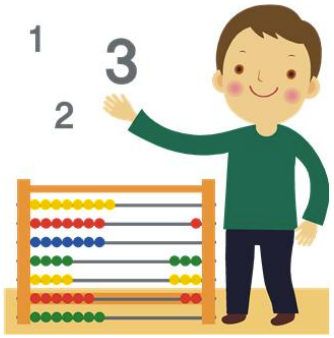
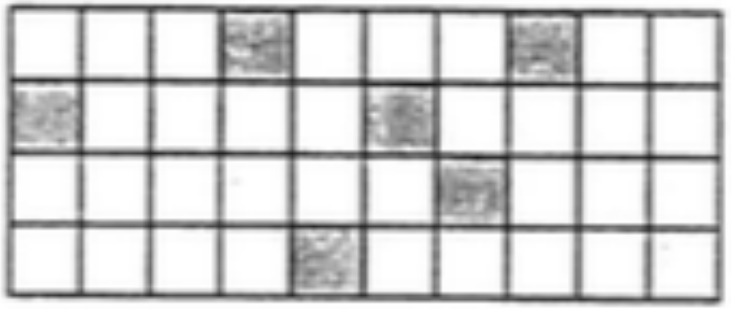


The Case of Ron Castleman

Two girls were explaining how they solved using the figure at the right:

6 x 2.5% = 15%

Since there are 40 squares in the diagram and the whole diagram needed to be 100%, each small square represents 2 ½ percent.



After the girls shared, Michael asked "How could something that wasn't subdivided into 100 equal 100%?"

Derrick added, "Are you saying that 100%=1? I thought 100%=100!"

From: Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development, pgs 37-63

Paint

Yellow and blue paint were mixed in a ratio of 5 to 3 to make green paint. After 14 liters of blue paint were added, the amount of yellow and blue paint in the mixture was equal. How much green paint was in the mixture at first?

Paint Solutions

What method did you use to solve?

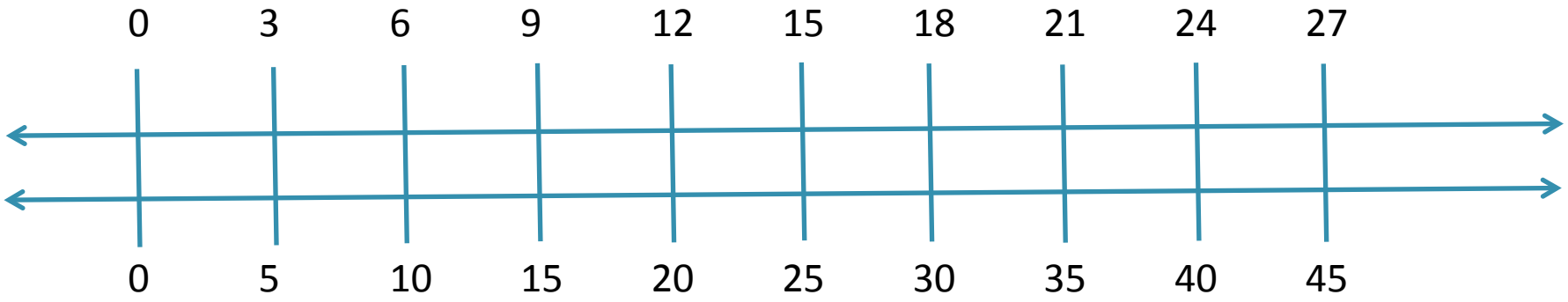
Paint Solutions - Tables

Liters of Yellow	Liters of Blue
5	3
10	6
15	9
20	12
25	15
30	18
35	21
40	24
45	27

Paint Solutions - Tables

Liters of Yellow	Liters of Blue	Difference
5	3	2
10	6	4
15	9	6
20	12	8
25	15	10
30	18	12
35	21	14
40	24	16
45	27	18

Double Number Lines



Tape Diagrams

- What are Tape Diagrams?
- How can a Tape Diagram be used to solve the Paint Problem?

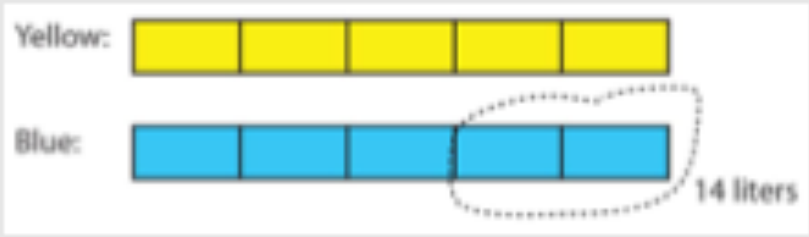
Relation to the Progressions

Representing a multi-step problem with two pairs of tape diagrams

Yellow and blue paint were mixed in a ratio of 5 to 3 to make green paint. After 14 liters of blue paint were added, the amount of yellow and blue paint in the mixture was equal. How much green paint was in the mixture at first?

At first:

Then:



2 parts $\longrightarrow$ 14 liters

1 part $\longrightarrow$ $14 \div 2 = 7$ liters

(original total) 8 parts $\longrightarrow$ $8 \cdot 7 = 56$ liters

There was 56 liters of green paint to start with.

This problem can be very challenging for sixth or seventh graders.

From: Progressions 6-7, Ratios and Proportional Reasoning, pg 7

Paint Solutions

Working Backward

I begin with equal amounts of blue and yellow paint.

I remove 14 liters of blue paint.

I have yellow and blue paint in a ratio of 5 to 3.

Equation

Let ***p*** be the number of liters of blue and yellow paint.

Now I have ***p*** liters of yellow paint and ***p - 14*** liters of blue paint.

This means: $\frac{p}{p-14} = \frac{5}{3}$

Paint Solutions

Scaling

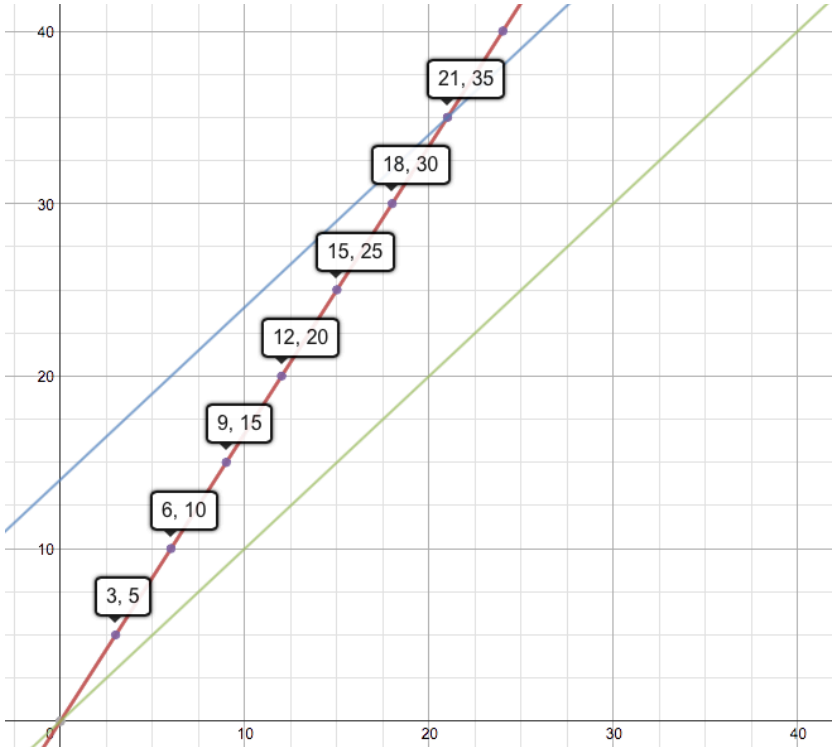
I have yellow and blue paint in a ratio of 5 to 3.

The difference between 5 and 3 is 2. I need that difference to be 14.

14 is 7 times as large as 2.

35 and 21 are 7 times as large as 5 and 3 respectively.

Graphing



Graphing by desmos.com

Progressions

- How can you use the Progressions documents to help you as an instructor?
- Should you only read the Progression for your grade level?

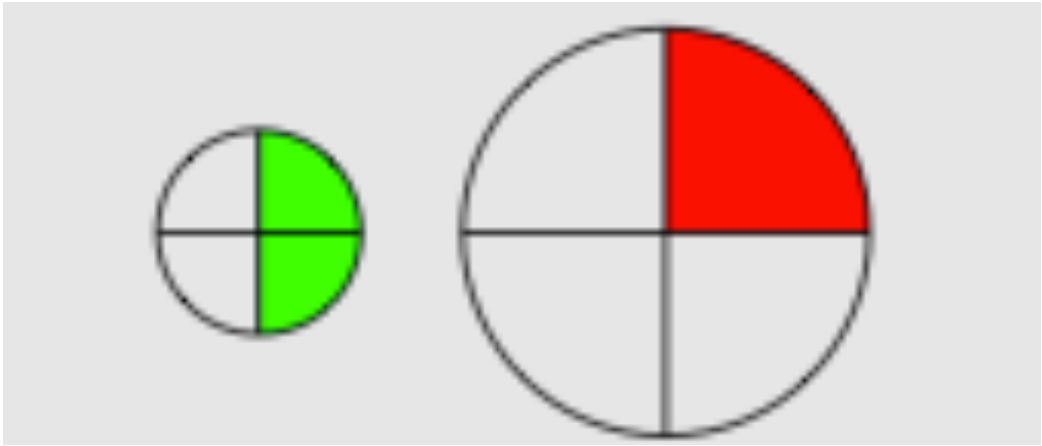
True or False?

½ is always greater than ¼.

Create a visual to defend your position.

True or False?

½ is always greater than ¼?



Progressions Documents

Browse through the different Progressions documents for various domains and grade levels.

Home People Programs Publications Events Visitors Resources

Progressions Documents for the Common Core Math Standards

Funded by the Brookhill Foundation

Progressions

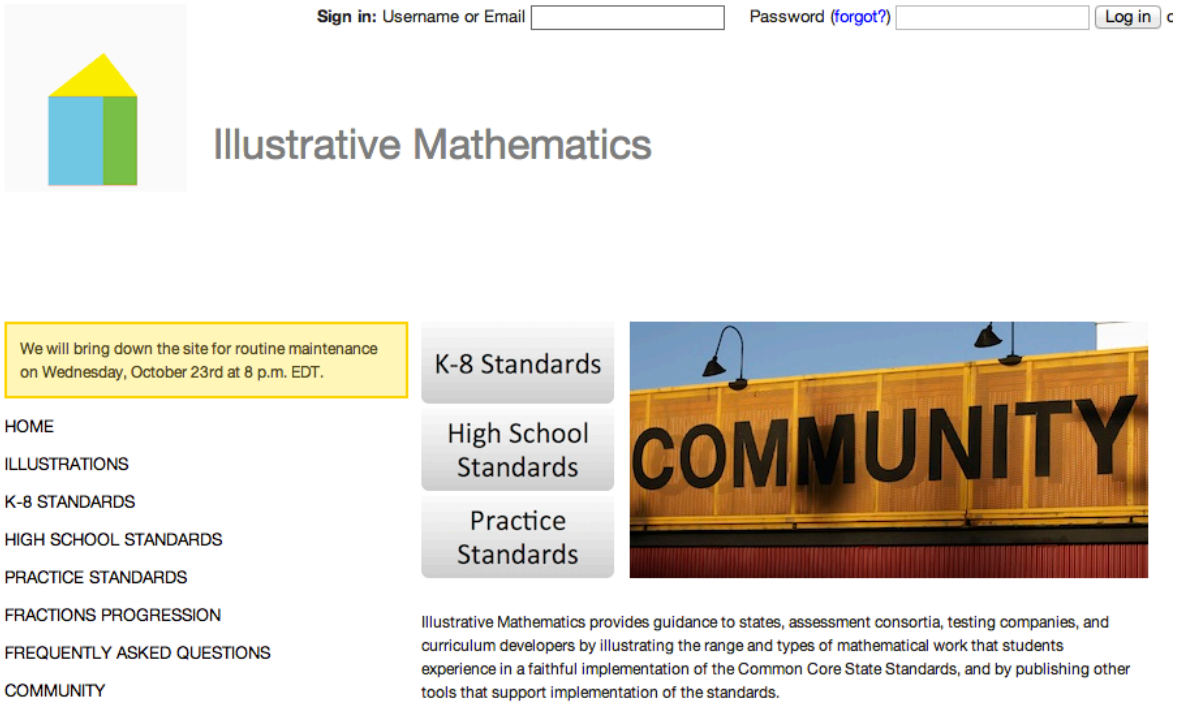
- Draft Front Matter
- Draft K–6 Progression on Geometry
- Draft K–5 Progression on Measurement and Data (measurement part)
- Draft K–5 progression on Measurement and Data (data part)
- Draft K–5 Progression on Number and Operations in Base Ten
- Draft K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking
- Draft 3–5 Progression on Number and Operations—Fractions
- Draft 6–8 Progression on Statistics and Probability
- Draft 6–8 Progression on Expressions and Equations
- Draft 6–8 Progression on The Number System; High School, Number
- Draft 6–7 Progression on Ratios and Proportional Relationships
- Draft High School Progression on Statistics and Probability
- Draft High School Progression on Algebra
- Draft High School Progression on Functions
- Draft High School Progression on Modeling

Progressions Documents for the Common Core Math Standards
Progressions
About this project
Working team

From: http://ime.math.arizona.edu/progressions/

Illustrative Mathematics Project

View the Standards, both content and process, and see examples of sample problems.



From: http://www.illustrativemathematics.org/

Some Other Resources

- Common Core Tools: http://commoncoretools.me/tools/
- Progression on Number and Operations – Fractions, 3-5: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf
- Progression on Ratios and Proportional Reasoning, 6-7: http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf
- Dan Meyer's Three Acts: http://threeacts.mrmeyer.com/

Thank you for attending!